

Polynomial Estimation of Fuel Consumption Coefficient Behaviour for Generation Expansion Planning and Cost Saving

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Abstract

Power system has several power plants each power plants has different generating units. At any point in time the total load in the system is met by the generating units, in different power plants. The polynomial-estimation techniques of fuel consumption pattern seek to run the power generating stations to obtain a maximum output of power demand with the aim of minimizing cost of operation, in a way to choose the best polynomial curves that can give an optimum savings. This paper present a systematic and simple formulation of cubic-quadratic polynomial equations, with associated cost of generations with some selected generating power station in Nigeria. The Matrix-Laboratory (MATLAB) soft-ware was adopted to obtain fuel – consumption – coefficient (α, β, λ) , which is represented as (x) , for purpose of presentation analysis in the program soft-ware. The techniques is formulated to address the challenging power generation problem facing the Nigeria power sector.

Keywords: Cost-Saving, Polynomial Estimation, Fuel Consumption, Coefficient Behaviour, Generation Expansion Planning, least-square technique

1. Introduction

The total generator operating cost (that is, production cost) includes: fuel-cost, labour-cost and maintenance cost. For purpose of analysis, variable-cost, is the only variable considered in this paper.

Fuel cost minimization requires knowledge of the fuel cost-curve for each of the generating units. An accurate representation of the cost-curve may requires a quadratic-polynomial form, or can be approximated in several ways with common one being:

- piece wise linear
- quadratic
- piece wise quadratic

Linear-approximation is not commonly used, while piece wise linear form is used in many production grade programming applications. Evidently a quadratic approximation is used in most non-linear programming applications.

2. Economic operation of power systems

Production cost of electricity is the most important economic factor in power system to be considered at all time. Therefore, it is a necessity to consider, all generating units combination to operate at their specific

load requirement or should be loaded in a manner to achieve optimum efficiency.

Hence the aim of economic-operation is to reduce the fuels-cost" in the power system, thereby ensuring reliability and efficiency at all time.

2.1 Economic Load Dispatch/Scheduling

This the determination of the generating output at different units in a power plant in a manner to minimize the total fuel-cost, and at the same time meet the total power demand.

2.2 Economic Load Dispatch

The economic load dispatch problem is the process of allocating loads demand to different generating units in a power plant at minimum fuel cost while considering the equality and inequality constraints conditions.

This evidently, become an optimization process of minimizing the total cost of all the generating unit committed to the load demand requirement.

2.3 Optimal Unit Commitment

The principle of engaging an equal incremental cost arrangement between generating-unit operations for load division, depends an experience in scheduling capacity allocation. In otherwise, the principle cannot specify the units which should be operated for a given, load demand, but rely strongly or deals with specifying the units which should be operated for a given load. This is because the total requirement of a power system varies throughout the day and reaches a different peak value from one day to another, the electricity utility has to decide in advance which generators to start-up and when to connect and commit it to the network, and the sequence in which the operating unit should be shut-down, which evidently become a computational problems for making such decisions, it is therefore geared to an effective unit commitment problems.

2.4 Methodology

Quadratic Polynomial Estimation

Case 1: Quadratic polynomial (ax^2):

The quadratic polynomial is an estimation technique to connect one, two, three or more points continuously.

For two points, like $p_1(x, y)$ and $p_2(x + h, y + k)$ on the curve of fig.1

$$y = ax^2 + bx + c \tag{1}$$

Where a, b, and c are constants then:

$$y + k = a(x + h)^2 + b(x + h) + c \tag{2}$$

Expansion equation (2):

$$y + k = ax^2 + 2ax + ah^2 + bx + bh + c \tag{3}$$

Recalling and equating equation (2) and (3):

That is,

$$y + k = ax^2 + 2ax + ah^2 + bx + bh + c \tag{3}$$

or

$$y = ax^2 + 0 + 0 + bx + 0 + c \tag{2}$$

Subtract (2) from (3):

$$k = 0 + 2ax + ah^2 + 0 + bh + 0 \tag{4}$$

or

$$k = 2ax + ah^2 + bh \tag{5}$$

Consider, the Cartesian, graph of y and x, when the gradient of the chord joining the two points and the limits of the gradient.

That is,

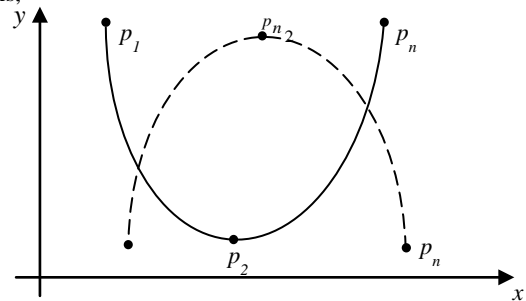


Fig 1. Quadratic polynomial with 2 points (and)

where: $x(h), y(k)$

If $\frac{h}{k}$ is the gradient, that is the ratio of:

$$\frac{x - step}{y - step} \tag{6}$$

$$\text{then, } \frac{k}{h} = \frac{2ax + ah^2 + bh}{h} = 2ax + ah + b \tag{7}$$

but as, h tends to zero,

$$h \longrightarrow 0$$

Then equation (7) becomes:

$$k = 2ax + ah^0 + b$$

or

$$\frac{k}{h} = 2ax + b \tag{8}$$

Similarly, in the case of cubic polynomial

If there are three (3) points: p_1, p_2 and p_3 as the polynomials connecting these points, which can be expressed as:

$$p_n : (x_n, y_n) \in R^2 \tag{9}$$

$$y = ax^2 + bx + c$$

The 1st order & 2nd order differential value of the quadratic polynomials are:

$$\frac{dy}{dx} = 2ax + b \tag{10}$$

and

$$\frac{d^2y}{dx^2} = 2a \tag{11}$$

Evidently, the curvature, k is obtained using (equation 1- 11) and can be expressed as:

$$k = \frac{2a}{\left(1 + (2ax + b)^2\right)^{3/2}} \tag{12}$$

Case 2: Cubic Polynomial (ax^3)

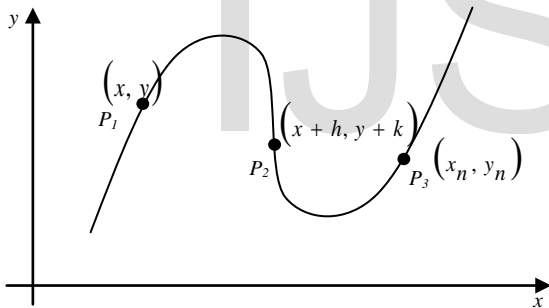


Fig 2: Graph of cubic polynomial with three (3) points

The cubic polynomial, of third-order function, can be expressed as:

$$p_n : (x_n, y_n) \in R^2 \tag{13}$$

or

$$y = ax^3 + bx^2 + cx + d \tag{14}$$

given $p_1(x, y)$, $p_2(x + h, y + k)$ and $p_3(y_n + k_n)$

On the curve,

$$y = ax^3 + bx^2 + cx + d \tag{14}$$

or for 1st – order differential equation:

$$\frac{dy}{dx} = 3ax^2 + 2bx + c \tag{15}$$

for 2nd – order differential equation:

$$\frac{d^2y}{dx^2} = 6ax + 2b \tag{16}$$

for $p(x, y)$, $p_2(x + h, y + k)$, $p_3(y_n + k_n)$

This means;

$$y + k = a(x + h)^3 + b(x + h)^2 + c(x + h) + d \tag{17}$$

but recall that:

$$(x + h)^3 = (x + h)(x + h)^2$$

Similarly:

$$(x + h)^3 = (x + h)(x^2 + 2xh + h^2)$$

or

$$(x + h)^3 = x^3 + 2x^2h + xh^2 + x^2h + h^3$$

$$(x + h)^3 = x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 \tag{18}$$

or

$$(x + h)^3 = x^3 + h^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

Similarly:

$$(x + h)^2 = x^2 + 2xh + h^2$$

Rewritten equation (17) and expanding:

$$y + k = a\left(x^3 + 3x^2h + 3xh^2 + h^3\right) + b\left(x^2 + 2xh + h^2\right) + C(x + h) + d \tag{18a}$$

or

$$y + k = ax^3 + 3ax^2h + 3axh^2 + ah^3 + bx^2 + 2bxh + bh^2 + cx + \frac{dy}{dx}h + d \quad 1^{st} \text{ order} \quad (24)$$

Recalled equation (14) and equating to equation (19) that is:

$$y = ax^3 + bx^2 + cx + d \quad (14)$$

Subtract (14) from (19):

$$K = 3ax^2h + 3axh^2 + ah^3 + 2bxh + bh^2 + ch \quad (20)$$

Where: $x(h)$, $y(k)$ are points on the curves

If $\frac{h}{k}$ is the gradient, that is the ratio: $\frac{x - step}{y - step}$

that is;

$$\frac{k}{h} = \frac{3ax^2h}{h} + \frac{3axh^2}{h} + \frac{ah^3}{h} + \frac{2bxh}{h} + \frac{bh^2}{h} + \frac{ch}{h}$$

or

$$\frac{k}{h} = 3ax^2 + 3axh + ah^2 + 2bx + bh + c \quad (21)$$

but as, $h \rightarrow 0$, then equation (21) becomes:

$$\frac{k}{h} = 3ax^2 + 3axh + ah^2 + 2bx + bh + c \quad (22)$$

or

$$\frac{k}{h} = 3ax^2 + 2bx + c$$

If $\frac{k}{h} = dy/dx = \text{constant} = 0$

if:

$$\frac{dy}{dx} = \frac{k}{h} = \text{gradient}$$

or

$$\frac{dx}{dy} = \frac{h}{k} = \text{gradient}$$

(23)

or

$$\frac{d^2y}{dx^2} = 6ax + 2b \quad (25)$$

Evidently, the curvature, k can be obtained, using the equation (1 – 24) and expressed as:

$$k = \frac{6ax + 2b}{\left(1 + (3ax^2 + 2bx + c)^2\right)^{3/2}} \quad (26)$$

2.5 Polynomial Curve Fitting for Generating Unit Capacity

nth degree polynomial of the form:

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n \quad (27)$$

This is fitted to these pair of data, then the following expression may be written to express the relationship between x and y .

$$y_1 = a_0 + a_1x_1 + a_2x_1^2 + a_3x_1^3 + \dots + a_nx_1^n \quad (28)$$

$$y_2 = a_0 + a_1x_2 + a_2x_2^2 + a_3x_2^3 + \dots + a_nx_2^n \quad (29)$$

$$y_3 = a_0 + a_1x_3 + a_2x_3^2 + a_3x_3^3 + \dots + a_nx_3^n \quad (30)$$

: : : :

$$y_n = a_0 + a_1x_n + a_2x_n^2 + a_3x_n^3 + \dots + a_nx_n^n \quad (31)$$

On summing up the column element, we have:

$$\sum_{i=1}^n y_i = na_0 + a_1 \sum_{i=1}^n x_i + a_2 \sum_{i=1}^n x_i^2 + a_3 \sum_{i=1}^n x_i^3 + \dots + a_n \sum_{i=1}^n x_i^n \quad (32)$$

The above equation forms the basis for the least – squares method for the polynomial curve fit.

– restructuring equation (32) into another form:

$$\sum_{i=1}^n y_i = n a_0 + a_1 \sum_{i=1}^n x_i + a_2 \sum_{i=1}^n x_i^2 \quad (33)$$

$$\sum_{i=1}^n y_i x_i = a_0 \sum_{i=1}^n x_i + a_1 \sum_{i=1}^n x_i^2 + a_2 \sum_{i=1}^n x_i^3 \quad (34)$$

$$\sum_{i=1}^n y_i x_i^2 = a_0 \sum_{i=1}^n x_i^2 + a_1 \sum_{i=1}^n x_i^3 + a_2 \sum_{i=1}^n x_i^4 \quad (35)$$

Into matrix – format:

$$\begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n y_i x_i \\ \sum_{i=1}^n y_i x_i^2 \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i^3 \\ \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i^3 & \sum_{i=1}^n x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \quad (36)$$

or

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i^3 \\ \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i^3 & \sum_{i=1}^n x_i^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n y_i x_i \\ \sum_{i=1}^n y_i x_i^2 \end{bmatrix} \quad (37)$$

or

where: $a_0 = \alpha$, $a_1 = \beta$, $a_2 = \gamma$

and

$$x_i = p(MW)$$

$$\sum y_i = \alpha n + \beta \sum p + \gamma \sum p^2 \quad (38)$$

$$\sum y_i p = \alpha \sum p + \beta \sum p^2 + \gamma \sum p^3 \quad (39)$$

$$\sum y_i p^2 = \alpha \sum p^2 + \beta \sum p^3 + \gamma \sum p^4 \quad (40)$$

or

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} n & \sum p & \sum p^2 \\ \sum p & \sum p^2 & \sum p^3 \\ \sum p^2 & \sum p^3 & \sum p^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum y_i \\ \sum y_i p \\ \sum y_i p^2 \end{bmatrix} \quad (41)$$

where:

y = the polynomial (cost) equation for each generating units, as:

$$y = \alpha + \beta p g + \gamma p g^2$$

α, β, γ are the fuel-consumption constant-coefficient

P_g : generating capacity

γ = fuel cost function

2.6 Some Data Collected from Different Generating Station in Nigeria to Estimate and Validate the Analysis Of Polynomial Equation and Fuel Consumption Pattern:

Table 1: Jebba Generating Power Unit

S/N	Power (MW)	Fuel Cost (\$/Hr) Y	P ² (Mw) ²	P ³ (Mw) ⁴	P ⁴ (MW) ⁴	P x Y(\$Mw/Hr)	P ² x Y(Mw) ² (S/Hr)
1	50	673.75	2500	125000	6250000	33687.5	1,684,375
2	75	928.4375	5625	421875	31640625	69632.812	5,222,460.938
3	100	1195	10000	1000000	100000000	119500	11,950,000
4	125	1473.4375	15625	1953125	244140625	184179.6875	23,022,460.94
5	150	1763.75	22500	3375000	506250000	2645625	39684375
6	175	2065.9375	30625	5359375	937890625	361539.0625	63,269,335.94
7	200	2380	4000	8000000	1600000000	476000	95,200,000
TOTAL	875	10480.312	126875	20234375	3426171875	1509101.562	240,033,007.8

Source: Central Bank of Nigeria (CBN), Statistical Bulletin

```
>> % analysis1: jebba generating power station
%authurs : braide s.l,d.c idoniboyeobu
program software: matrix laboratory (matlab)
% determination fuel consumption coefficient,x( alpha,
betaand gamma)
A=[7,875,126875;875,126875,20234375;126875,2023
4375,3426171875];
B=[10480.3125;1509101.563;240033007.8];
x=inv(A)*B
x =
199.9999
9.0000
0.0095
∴ y = 200+9P +0.0095 P2
```

Table 2: Kainji Generating Power Unit

S/N	Power (MW)	Fuel Cost (\$/Hr) Y	P ² (Mw) ²	P ³ (Mw) ⁴	P ⁴ (MW) ⁴	P x Y (\$Mw/Hr)	P ² x Y (Mw) ² (S/Hr)
1.	80	733.6	6400	512000	40960000	58688	4695040
2.	100	880	10000	1000000	100000000	88000	8800000
3.	150	1277.5	22500	3375000	506250000	191625	28743750
4.	200	1720	40000	8000000	1600000000	344000	68800000
5.	250	2207.5	62500	15625000	3906250000	551875	137968750
6.	300	2740	90000	27000000	8100000000	822000	246600000
7.	350	3317.5	122500	42875000	15006250000	1161125	406393750
TOTAL	1430	12876.1	353900	98387000	29259710000	3217313	902001290

Source: Central Bank of Nigeria (CBN), Statistical Bulletin

```
>> % analysis 2: kainji generating power station
%authurs: braide s.l, d.c idoniboyeobu
program software: matrix laboratory (matlab)
% determination of fuel consumption coefficient,
x(alpha, beta and gamma)
>>
A=[7,1430,353900;1430,353900,9838700;353900,983
87000,29259710000];
>> B=[12876.1;3217313;902001290];
x=inv(A)*B
x =
-122.8625
9.5852
0.0001
>>P
∴ y = -123+10P +0.0001 P2
```

Table 3: Afam Generating Power Unit

S/N	Power (MW)	Fuel Cost (\$/Hr) Y	P ² (Mw) ²	P ³ (Mw) ⁴	P ⁴ (MW) ⁴	P x Y (\$Mw/Hr)	P ² x Y (Mw) ² (S/Hr)
1.	50	730	2500	125000	6250000	36500	1825000
2.	75	1000	5625	421875	31640625	75000	5625000
3.	100	1280	10000	1000000	100000000	128000	12800000
4.	125	1570	15625	1953125	244140625	196250	24531250
5.	[150	1870	22500	3375000	506250000	280500	42075000
6.	175	2180	30625	5359375	937890625	381500	66762500
7.	200	2500	40000	8000000	1600000000	500000	11200000
TOTAL	875	11130	126875	20234375	3426171875	1597750	164818750

Source: Central Bank of Nigeria (CBN), Statistical Bulletin

```
>> % analysis 3: afam generating power station
%authurs: braide s.l, d.c idoniboyeobu
```

```

program software: matrix laboratory (matlab)
% determination of fuel consumption coefficient,
x(alpha, beta and gamma)
>>
A=[7,875,126875;875,126875,20234375;1268,202343
75,3426171875];
>> B=[11130;1597750;164818750];
>> x=inv(A)*B
x =
    716.8561
    0.3361
    0.0459
>>
∴ y = 717+0.3361P +0.0459 P2
    
```

Table 4: Sapele Generating Power Unit

S/N	Power (MW)	Fuel Cost (\$/Hr) Y	P ² (Mw) ²	P ³ (Mw) ⁴	P ⁴ (MW) ⁴	P x Y (\$Mw/Hr)	P ² x Y (Mw) ² (S/Hr)
1.	50	858.75	2500	125000	6250000	42937.5	2146875
2.	60	997	3600	216000	12960000	59820	3589200
3.	70	1136.75	4900	343000	24010000	79572.5	5570075
4.	80	1278	6400	512000	40960000	102240	8179200
5.	90	1420.75	8100	729000	65610000	127867.5	11508075
6.	100	1565	10000	1000000	100000000	156500	15650000
7.	110	1710.75	12100	1331000	146410000	188182.5	20700075
8.	120	1858	14400	1728000	207360000	222960	2675520
TOTAL	680	10825	62000	5984000	603560000	980080	94098700

Source: Central Bank of Nigeria (CBN), Statistical Bulletin

```

>> %analysis 4: sapele generating station
%authurs :braide s.l.d.c idoniboyeobu
program software: matrix laboratory (matlab)
% determination of fuel consumption coefficient,x(alpha,
betaand gamma)
A=[8,680,62000;680,62000,5984000;62000,5984000,6
03560000];
B=[10825;980080;94098700];
>> x=inv(A)*B
x =
    190.0000
    13.0000
    0.0075
>>
∴ y = 190+13P +0.0075 P2
    
```

Table 5: Egbin Generating Power Unit

S/N	Power (MW)	Fuel Cost (\$/Hr) Y	P ² (Mw) ²	P ³ (Mw) ⁴	P ⁴ (MW) ⁴	P x Y (\$Mw/Hr)	P ² x Y (Mw) ² (S/Hr)
1	50	772.5	2500	125000	6250000	38625	1931250
2	75	1075.65	5625	421875	31640625	80673.75	6050531.25
3	100	1390	10000	1000000	100000000	139000	13900000
4	125	1715.625	15625	1953125	244140625	214453.125	26806640.63
5	150	2052.5	22500	3375000	506250000	307875	46181250
TOTAL	500	7006.275	56250	6875000	888281250	780626.875	94869671.88

Source: Central Bank of Nigeria (CBN), Statistical Bulletin

```

>> %analysis 5: egbin generating power station
>> % athurs:braide s.l, d.c idoniboyeobu
    
```

```

program software: matrix laboratory (matlab)
>> %determination of fuel consumption coefficient,
x(alpha, beta and gamma)
>>
A=[5,500,56250;500,56250,6875000;56250,6875000,8
88281250];
>> B=[7006.275;780626.875;94869671.88];

>> x=inv(A)*B
x =
    199.9900
    11.0005
    0.0090
>>
∴ y = 200+11P +0.0090 P2
    
```

Table 6: Shiroro Generating Power Unit

S/N	Power (MW)	Fuel Cost (\$/Hr) Y	P ² (Mw) ²	P ³ (Mw) ³	P ⁴ (MW) ⁴	P x Y (\$Mw/Hr)	P ² x Y (Mw) ² (S/Hr)
1.	100	710	10000	1000000	100000000	71000	7100000
2.	150	997.5	22500	3375000	506,250,000	149625	22443750
3.	200	1320	40000	8000000	1600000000	264000	52800000
4.	250	1677.5	62500	15625000	3906250000	419375	104843750
5.	300	2070	90000	27000000	8100000000	621000	186300000
6.	350	2497.5	122500	42875000	15006250000	874125	305943750
7.	400	2960	160000	64000000	25600000000	1184000	473600000
8.	450	3457.5	202500	91125000	41006250000	1555875	700143750
9.	500	3990	250000	125000000	62500000000	1995000	997500000
TOTAL	2700	19680	960000	378000000	1.58325E+11	7134000	2850675000

Source: Central Bank of Nigeria (CBN), Statistical Bulletin

```

>> %analysis 6: shiroro generating power station
% athurs:braide s.l, d.c idoniboyeobu
%determination of fuel consumption coefficient,
x(alpha, beta and gamma)
>>
A=[9,2700,960000;2700,960000,378000000;960000,3
78000000,158325000000];
>> B=[19680;7134000;2850675000];
>> x=inv(A)*B
    
```

x =
 240.0000
 4.0000
 0.0070

>>
 ∴ y = 240+4P +0.0070 P²

From our formation of matrix recalled equation 41:

$$\begin{bmatrix} \alpha \\ \beta \\ y \end{bmatrix} = \begin{bmatrix} n & \sum p & \sum p^2 \\ \sum p & \sum p^2 & \sum p^3 \\ \sum p^2 & \sum p^3 & \sum p^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum y_i \\ \sum y_i p \\ \sum y_i p^2 \end{bmatrix} \quad (41)$$

Now, Inputing the computation data into equation (38, 39, 40 and 41) respectively.

We have:

$$\sum y_i = \alpha \cdot n + \beta \sum p + y \sum p^2 \quad (38)$$

$$\sum y_i p = \alpha \sum p + \beta \sum p^2 + y \sum p^3 \quad (39)$$

$$\sum y_i p^2 = \alpha \sum p^2 + \beta \sum_{n=7} p^3 + y \sum p^4 \quad (40)$$

Thus, from least-square regression for jebba generating station:

$$10480.3125 = 7\alpha + 875\beta + 126875\gamma \quad (42)$$

$$150919.562 = 875\alpha + 126875\beta + 20234375\gamma \quad (43)$$

$$240436475.3 = 126875\alpha + 20234375\beta + 3426171875\gamma \quad (44)$$

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 7 & 875 & 126875 \\ 875 & 126875 & 20234375 \\ 126875 & 20234375 & 3426171875 \end{bmatrix}^{-1} \begin{bmatrix} 10480.3125 \\ 150919.562 \\ 240436475.3 \end{bmatrix}$$

where $\alpha = 200, \beta = 9, \gamma = 0.0095$

Therefore, $\therefore y = 200+9P +0.0095 P^2$

Conclusion

Application of polynomial method have been in existence for some time. Different studies have been employed for solution for fuel cost estimation. An efficient and simple technique is used to solve the above problems. The result shows the different cost function equation (y) which indicates the cost-function equation for each of the generating station, which are determined. In order words, the system are monitored to adjust the fuel coefficient to obtain an optimal cost-saving, because the fuel coefficient (α, β, γ) to a large extent determined the generating consumption pattern thereby making the operating capacity optimally, therefore with careful adjustment of the fuel coefficient will save a lot of money annually and will strongly improve the stability of the power system.

References

1. Kumoko, D. T. (2011). "Minimization of electric power generation cost" A case study of kolo-creek gas turbine station. pp 10-23
2. Mahta, V. K. & Rohit Mahta (2013). "Principle of power station" S.chand & Company Pvt. Ltd. Ram Nager. New Delhi-110055. pp. 9-10.
3. Mary, B., Richard, P.O. & Anya, C. (2012). "History of Optimal power flow and formation" pp. 7-9
4. Momoh, J. A., Guo, S. X., Ogbuobiri, E. C., & Adapa, R., (1994). "The Quadratic Interior Point Method Solving Power System Optimization Problem", IEEE Transaction on Power System, Vol. 9 pp. 1327-1336
5. Sheble, G., & M, Calley, J., (2013). ("Module E3; Economic dispatch

calculation") used in EE303 at Iowa state university, pp. 48-51

6. Sood, Y. R., (2007) "Evolutionary Programming Based Optimal Power Flow & Its Validation for Deregulation Power System Analysis" Electric Power & Energy System, vol. 29, pp. 262-272
7. Wood, A. J. & Wollenberg, B.F., (1996a) "Power Generation Operation & Control," second edition, John Wiley & Sons, New York, NY. pp. 154
8. Wood, A. J. & Wollenberg, B.F., (1996a) "Power Generation Operation & Control," New York, NY: John Wiley & Sons, Inc. pp 39,517
9. Yuryevich, T. & Wong, K. P. (1999). "Evolutionary Programming Based Optimal Power Flow Algorithm". IEEE Transaction on power system, Vol. 14 pp. 1245-1250.